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Deck Watchkeeper

Mathematics and Physics

E-CAMPUS AUSTRALIA PTY LTD T/A ECA MARITIME COLLEGE

Contents

Algebra

In Arithmetic all numbers are expressed in terms of the digit, all of which have definite values. In Algebra, as well as these digits, symbols which usually have no single value are used. Algebra includes all the definitions and method a of arithmetic, so that signs +, -, x, ÷, =, for example, have the same meaning as arithmetic, but these definitions and methods are extended and applied in wider and more general uses, and not only to ordinary numbers but to quantities which are not found in arithmetic. It will be found that in the early stages the rules and processes of algebra follow much the same pattern as those for arithmetic.

Before proceeding to investigate the functions of the trigonometrically ratios and the solution of triangles, it is necessary to be familiar with the following signs, symbols and abbreviations:

a-b means the value of **"b"** us to be subtracted from the value of **"a".** When this sign is used the large quantity must be put down first, as it is impossible to take a large quantity from a smaller one. If this is not done the result will be a negative quantity; thus if we have **6-3** we know the remainder will be **3**; but is we have **3-6 the result will be in the negative 3, that is, 3-6=-3.**

a~b – This sign means the difference between **"a" and "b"**, and is preferential to the minus sign, inasmuch that it is quite immaterial as to the order in which the quantities are written down. The larger or smaller quantity may be put in the first position, **thus a~b = b~a.**

a^2- The result of "a" multiplied by "a"

√a – A quantity, which being multiplied by itself, produces "a", i.e. the square roof of "a".

(a+b) (a+b) – The sum of "a" and "b" multiplied b the sum of "a" and "b", the x is omitted for brevity. It could also be written $(a+b)^2$

$\frac{b}{c}$ is a fraction. It means that "b" is to be divided by "c" to obtain the value of the expression.

If "b" is greater that "c" the result will be more that 1; in both cases it gives us the proportion or ratio that "b" bears to "c", whether larger or smaller. It is therefore, the most convenient method of writing down the proportion of ratio that exits between two quantities.

The value of the expression ½ means the proportion that 1 is to 2, which is a half. The value of the fraction 3/2 is as 3 is to 2, which is one and a half. The upper value is called the numerator and the lower part of the denominator. In any fraction, we can multiply or divide the numerator and denominator by the same amount without altering the value of the fraction.

Thus

$1/4 = (1 \text{ x } 4)/(4 \text{ x } 4) = 4/16 = ((4/2)/16)/2 = 2/8 = 1/4$

$a/2$ - A divided by 2, or the half of "a"

$(a + b + c)/2$ – Half the sum of "a" and "b" and "c"

$\sqrt{2^2}$ - The square root of 2 multiplied by the square root of 2, which is 2, $\sqrt{2^2}$ = 2, for 2 x 2 = 4, and $\sqrt{4}$ = 2. Similarly, $\sqrt{a^2}$ = a.

$\sqrt{\frac{1}{4}}$ - The square root of 1 divided by the square root of 4; this becomes

$\sqrt{\frac{1}{4}}$ = 1/2 , for the $\sqrt{1}$ = 1, and $\sqrt{4}$ = 2

$\left(\frac{1}{2}\right)^2$ is 1^2 divided by 2^2, this becomes $1^2/2^2$ = ¼

The root sing $\sqrt{}$ is used in evolution, which means the lowering of quantities from higher to lower powers; for example $\sqrt[3]{8}$ = 2, that is to say the cube root of 8 is 2.

Involution (Power)

This expression means the raiding of a quantity to a higher power, this being the reverse to Evolution.

A x a x a = a^3 - a is the base and 3 is the index or exponent.

3 x 3 = 3^2 = 9 – Here 3 is the base, 2 is the index or exponent and 9 is the power.

The notation used for expressing powers is: 3 x 3 = 3^2

6 x 6 x 6 = 6^3

a x a x a x a = a^4

The figure which indicates the number of factors multiplied together is called the index of the power. Hence in 3^3, 6^6, and a^4 the numbers 2, 3, 4 are called indices.

Coefficient

The coefficient of a term may be a number proceeding is, as in 2x, where 2 is the coefficient of x. The expression is equivalent to a+x.

The coefficient, however, is not always a number e.g. in the expression ax, a is the coefficient of x. It may also take a compound form as in 2ax, where 2a is the coefficient of x. The expression might be written as (a+a)x.

Algebraic Expression

A collection of numbers and symbols connected by the signs +, -, x, and ÷ is called an algebraic expression

Term

Any combination of numerals and symbols connected by the + and – signs is called a Term. **(Note: the signs x and ÷ DO NOT separate terms.)**

$$\pi r^2, 2ab, \frac{1}{x}, are\ each\ simple\ terms$$

An expression may contain more than one term:

$2a + 4a^2$ is a binomial expression (it contains 2 terms)

$x^2 + 4xy - y^2$ is a trinomial expression (it contains 3 terms)

multinomial expressions consist of more than 3 terms.

Note: x or ÷ do not separate terms:

$5a^2 \times 3ax$ is one term; $5a^2 \times 3ax + 4a$ contains 2 terms. $\underline{5a^2 \times 3\,ax}$, and $\underline{4a}$

Like terms are those that differ only in the numerical coefficient.

6a, 9a, and – 6a	are like terms
$2x^2z$, $7x^2y$ and - x^2y	are like terms
2x, 3ax, and $4x^2$ -	are unlike terms

Like terms can be added and subtracted as in mathematics:

$$8a + 7b - 6C - 2a + b + 10c$$

Collect like terms together = $\underline{8a - 2}$ + $\underline{7b+b}$ – $\underline{6c + 10C}$ = 6a + 8b + 4c

Addition

When adding together fractions it is essential that the denominators, the figure below the line in the equation, are the same. This allows you to operate with equivalent units.

When the denominators are the same for all of the fractions the process is simply to add together the numerators for each of the respective fractions, (the value above the line).

Example: $\frac{1}{7}+\frac{3}{7}+\frac{4}{7}=\frac{8}{7}$

$\frac{8}{7}$ is an <u>**improper**</u> fraction because the numerator is greater than the denominator. This improper fraction needs to be converted to a proper fraction.

To convert to a proper fraction divide the numerator by the denominator.

In this case 8 is divided by 7. As $\frac{7}{7} =$ 1, then the whole the fraction $\frac{8}{7}$ becomes 1 $\frac{1}{7}$.

When the denominators for each individual fraction are not the same and you still want to add them together, you need to identify the **lowest common denominator.**

The lowest common denominator is the lowest possible number that all of the existing denominators will divide into. Once you have found the lowest common denominator, you will then convert the fractions to this format.

To find the lowest common denominator follow these few steps.

1. Multiply the denominators of the individual fractions to find a common denominator.
2. Reduce the common denominator to the lowest number that all the original denominators will divide into.
3. Multiply each numerator by same amount its denominator was multiplied by when determining the lowest common denominator.
4. Add together all of the numerators and present the answer in its simplest form.

Example: $\frac{3}{8}+\frac{1}{10}+\frac{2}{5}$

Step 1. Common denominator = 8 x 10 x 5 =400

Step 2. Lowest common denominator = 40
(8, 10 and 5 will all divide into 40).

Step 3. Having found the lowest common denominator, each fraction in the example can now be thought of in another way:

$$\frac{3}{8} x \frac{5}{5} = \frac{15}{40} \qquad \frac{1}{10} x \frac{4}{4} = \frac{4}{40} \qquad \frac{2}{5} x \frac{8}{8} = \frac{16}{40}$$

Step 4. Similarly, when reducing a fraction to its simplest form following addition.

$$\frac{15}{40} + \frac{4}{40} + \frac{16}{40} = \frac{35}{40}$$

$\frac{35}{40}$ is not the simplest form of the fraction because 5 will divide exactly into both the numerator and the denominator.

Now divide numerator and denominator by 5.

Therefore: $\frac{35}{40} = \frac{7}{8}$

Brackets

When a bracket is immediately preceded by a + sign, the bracket may be removed without altering the signs of the expressions.

When a bracket is immediately preceded by a – sign, the sign of every term must be changed here when the bracket is removed. This is readily demonstrated as follows;

1. (8+4) + (7-3) = 8 + 4 + 7 – 3 -= 19 – 3 = 16. None of the signs have been changed her as each pair of brackets was preceded by a + sign.
2. (8+4) – (7-3) = -8-4-7+3= -19+3 = -16, All the signs have been changed here as each bracketed quantity was preceded by a minus sign(-)

Remove the bracket and simplify the following expressions:

1. (4a+3b-c) – (2a+b+3c) = 4a + 3b- c – 2a – b – 3c
2. –(3x + 5y-4z) + (5x – 3y – 10z) = - 3x – 5y + 4z + 5x – 3y – 10z

When brackets are preceded by numbers each term inside the bracket is multiplied by the number.

Example: 2(a+b+c) – 2(a+b÷c)

Each term inside the first pair of brackets is multiplied by 2; each term inside the second pair is multiplied by – 2, and we obtain

2a ÷ 2b + 2c – 2a – 2b – 2c = 0 (because the plus and minus quantities are equals)

Insertion of Brackets

In algebraic operations it is very often necessary to enclose groups of terms within brackets.

If the sign immediately in front of the bracket is + make no change in the signs of the term embraced by the bracket; but if the sign is – then change the sign of every term inside the bracket.

Expressions can be changed by using brackets:

3a + 3ay = 3a (1+y)

$4a^2x$ – 2ay = 2a (2ax – y)

The expression a + b – c – d could be arranged thus: (a+b) – (c+d). Note here that it was necessary to change the signs of c and d because the brackets enclosing them were preceded by a minus sign. In the expression’s ab + ac + ad, a is a factor common to each term; therefore a can be divided inti each of the values b, c and d. The expression could therefore be arranged thus: **a(b+c+d)**

Enclose the following expressions in brackets taking out the common factors; b is the common factor in the 1st two terms and D is the common factor in the second term.

1. ab+bc+cd-de = b(a+c) + d (c -e)
2. a^2 - ac - 2abc - ab^2 = a (a-c) - ab (2c+b)
   = a [(a-c) – b(2c+b)

Substitution

When two expressions are equal for all values on the unknown, they form an identity.

5x – 4 + 3x + 8 = 8x + 4

If the two expressions are equal only for certain values of the unknown, then the relation is any equation.

6x + 8 = 5x + 10 only when x = 2

An equation is simply a statement that two expressions are equal to one another.

The equation, a = 5b, where b=3, can be solved by the method of "substitution, i.e. by substituting for b in the equation the value of 3. We now have a = 5 x 3 = 15

Example a = 3b +5, when b = 2

a = 3 x 2 + 5 =

6 + 5 = 11

Exercise 2

1. $x^2 = 4y^2 + 3y + 5$, when y = 4
2. $2x^2 = 3y^2 + 2\sqrt{y} + 39$, when y = 9
3. If a = 7, b = 4 and c = 2, evaluation the following
   a. $a^2 (a-b) + b^2 (b-c) + c^2(a-c)$
   b. Bc{($a^2 - c^2$) – 2 (b +2c)}