Mathematics for Officers in Charge of a Navigational Watch

ECA MARITIME 4/3/2024

FOR USE WITH MAR50320 - DIPLOMA OF MARITIME OPERATIONS

Outcome of Course

- This course is designed to address the underlying competencies required for completion of the IMO model 7.03 course for Officers in Charge of a Navigational Watch.
- It will address STCW requirements II/1 and II/2 for mathematical knowledge required for officers, chief mates and masters in charge of a navigational watch.
- Upon completion you should be in possession of the fundamental knowledge required to complete topics later covered in this course.

Topics

- Algebra
- Graphs
- Interpolation, Variation and Proportion
- Geometry
- Trigonometry
- Mensuration
- Spherical Triangle
- Vectors
- Circle, Ellipse and Hyperbola
- Statistics

1.1 Algebra

- Using variables and constants to show mathematical processes is how simple algebraic expressions are made.

- When making expressions or simplifying them, remember the order of processes (PEMDAS/BODMAS). To do this, start with operations inside parentheses, then move on to exponents (powers and roots), then multiply and divide (from left to right), and finally add and remove (from left to right).

1.1 Algebra: Simple Expressions

- To express a sum, you simply use the addition (+) operator between variables, constants, or a combination of both.

    Adding 5 to X can be expressed as X + 5

- Differences are expressed using the subtraction (-) operator.

    Subtracting 5 from X can be expressed as X - 5

- Products involve the multiplication (*) operator, but in algebraic expressions, the operator is often omitted and implied by placing the variables or constants next to each other.

    X multiplied by 4 can be expressed as 4X

- Quotients use the division (/) operator or are represented by a fraction bar.

    X divided by 3 can be expressed as X/3

1.1 Algebra: Expanding Fractions

- Expanding out fractions allow us to break a sophisticated operation into its component parts to assist in simplification or solving an equation.
- Most of the time, expanding fractions means making the denominator bigger while leaving the value of the fraction the same.

    2/3 can be expanded out into 8/12

- This method is also called "finding equivalent fractions." The goal is to multiply the fraction's numerator (the number at the top) and denominator (the number at the bottom) by the same non-zero number.

    2/3 + 5/6 + 3/4 can be expanded into 12’s to assist solving

    8/12 + 10/12 + 9/12 = 27/12 **or** 2 3/12

1.1 Algebra: Expanding Fractions

- To expand out bracketed fractions, you have to spread the terms outside the brackets over each term inside the brackets.
- Often, algebraic formulas are used in this step, which is a bit different from just expanding fractions. Let's use cases to walk you through the steps.
- The key principle here is to affect all items inside the bracket by the factor outside the bracket.

A(B+C-D) is equal to AB+AC-AD

1.1 Algebra: Simplifying Fractions

- One important part of mathematics is simplifying expressions, which means taking them down to their simplest form to make them easier to understand or change in other ways.
- Do things one step at a time to avoid making mistakes. Look for opportunities to factor by factoring polynomials or using special products.

1.1 Algebra: Solving Fractions

- To solve problems with linear equations and simultaneous linear equations, you need to figure out what the unknown factors are, use the problem description to set up equations, and then solve these equations to find the unknowns' values. For both kinds of problems, let's break the process down into steps.
- When there are two or more equations with two or more factors in simultaneous linear equations the goal is to find the variable values that make all the equations true at the same time.
- Most of the time replacement, elimination, and matrices are used (for more complicated systems). We are going to work on replacing and getting rid of.

1.1 Algebra: Solving Equations

- To solve problems with linear equations you need to figure out what the unknown factors are, use the problem description to set up equations and then solve these equations to find the unknowns' values. Let's break the process down into steps.
- Most of the time we can solve equations by using the elimination technique:
  - Arrange the equations with like terms (expand and simplify).
  - Multiply one or both equations by constants if necessary to get coefficients of one variable to be the same.
  - Add or subtract the equations to eliminate one variable.
  - Solve the resulting equation for the remaining variable.
  - Substitute this value back into one of the original equations to find the other variable.

1.1 Algebra: Solving Fractions Example

Solve (X-15)x8 = 8 to find X

First lets expand out the bracket:

(X-15) x 8 = 8 becomes 8X – 120 = 8

Now Imagine both sides of the equal sides are two mirror reflections; what happens to one happens to the other.

8X – 120 = 8 …add 120 to both sides to simplify.

8X = 128 ….divide both sides by 8 to get X

X = 16

1.1 Algebra: Solving Fractions Example

Solve two simultaneous equations:

$2X + 4y = 6$

$5X - 2y = 12$

Choose either the variable with the differing sign and multiply one equation to render equivalent in value to the other.

$2X + 4y = 6$

$10X - 4y = 24$

Now add both sides of both equations to the other:

$12X = 30$ …. There for $X = 30/12 = 2.5$

1.1 Algebra: Quadratic Equations

- Quadratic equations are equations in the form of $AX^2 + BX + C = 0$. Where A does not equal 0.
- Solving them is an essential skill in Algebra.
- You can solve quadratic equations using factoring by breaking the equation into two brackets.

  Solve $X^2 - 8X + 8 = 0$

  $(X – 4)(X – 4) = 0$

  X = 4 **or** 4

- **Quadratic equations are solved for one of two answers.**

1.1 Algebra: Quadratic Equations

Quadratic Formula: For $ax^2 + bx + c = 0$, the solutions for x are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So for $3X^2 + 5X + 2 = 0$

X = [- 5 +/- √(5^2 – 4(3x2))] / 2 x 3
X = [-5 +/- √ 1] / 6
X = -0.67 **or** -1

1.1 Algebra: Absolute Error

- Absolute error is a way to measure how accurate measurements, calculations, and approximations are in many areas, such as mathematics, physics, engineering, and statistics.
- This is very important for knowing the limits of measuring tools and methods and for making sure that testing and computer results are more accurate.
- To calculate absolute error you simply determine the difference between the calculated and observed error and express it as a positive number.

For example a calculated value of 4 meters and an observed value of 3.98 would result in an absolute error of 0.02 meters.

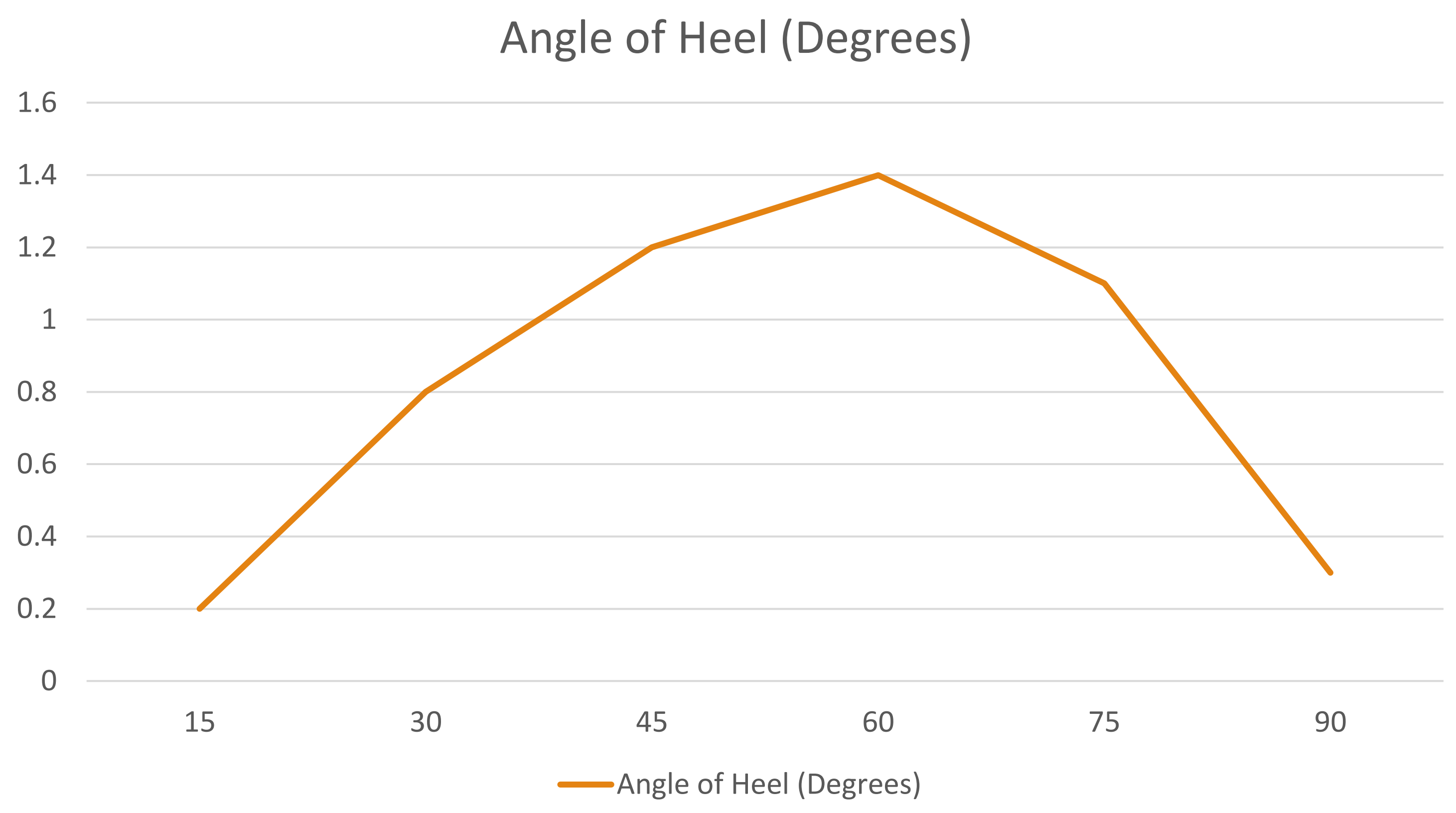
1.1 Algebra: Relative Error

- A relative error measure lets you see how big the exact error is compared to the real value of a problem.
- It gives you an idea of how big the error is compared to the amount being measured or calculated.
- This comparison gives you a better idea of how accurate a measurement or figure is because it considers the fact that an absolute error can be bigger for a small amount than for a big one.
- It is usually given as a percentage to show how big the error is compared to the whole true value.
- Relative error is equal to Absolute Error / True Measurement.

1.2 Graphs

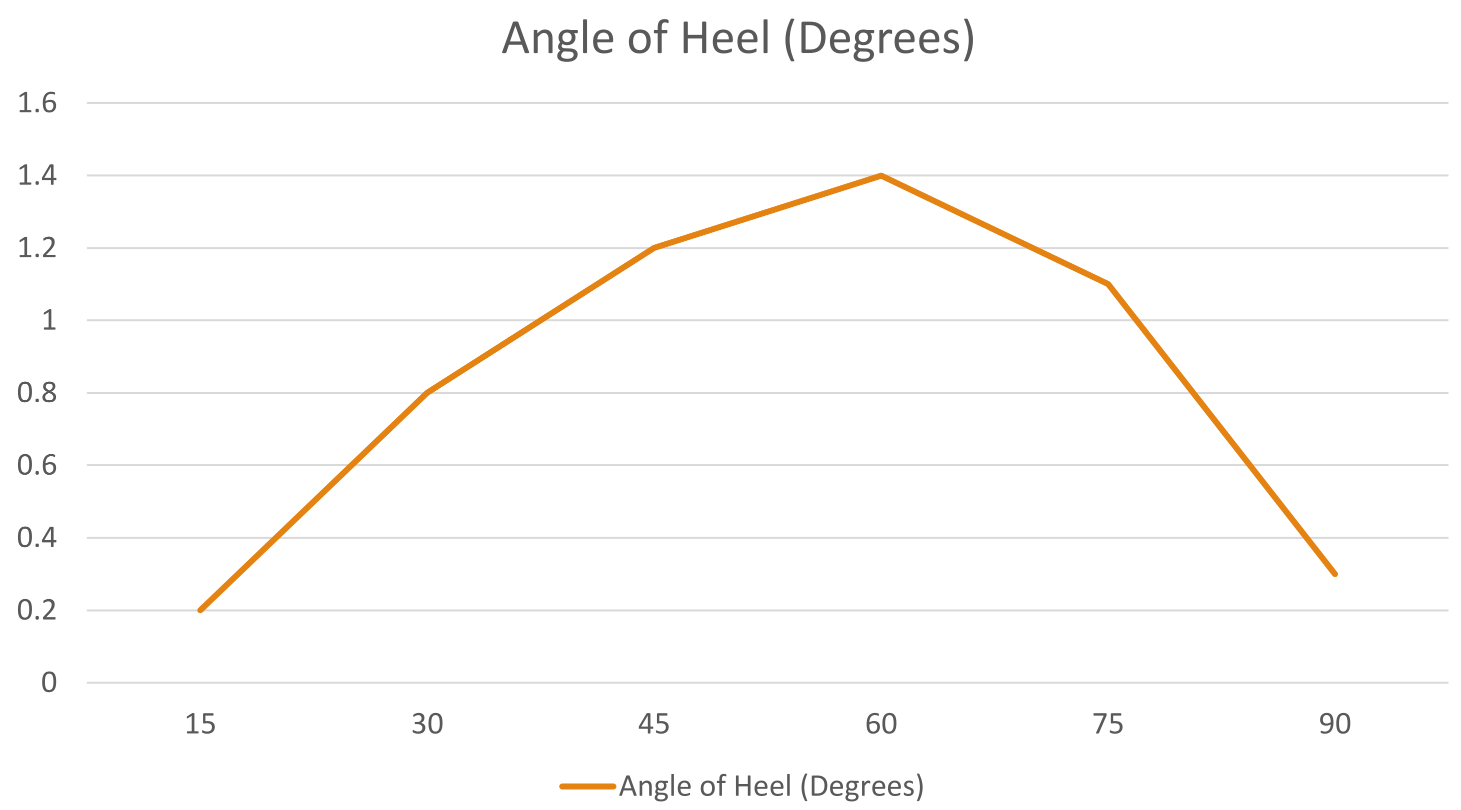
- A graph is a picture that shows data or the mathematical links between variables.

- Graphs are made up of points, which are called **vertices or nodes**, and lines or curves, which are called **edges or arcs**, that connect them.



1.2 Graphs

- A graph has two Axis: X (horizontal) and Y (vertical).
- These axis should be assigned to two related attributes.
- The scale of these axis should be the largest possible while encompassing all data points.



1.2 Graphs: Cartesian Co-ordinates

- You can use numbers called coordinates to show points in a two-dimensional plane.
- They are called Cartesian coordinates, after the French scientist and philosopher René Descartes.
- An ordered pair of numbers (x, y) tells you where a point is in space in relation to the origin point (where X and Y equal 0).
- “X" tells you where the point is horizontally (the abscissa) and "y" tells you where it is vertically (the ordinate).

1.2 Graphs: Plotting Graphs

- To plot a graph for use in the maritime industry you must first accumulate a series of datapoints as ordered pairs.
- These datapoints are then plotted onto the graph.
- Once plotted these points are joined together by straight or curved lines depending on it's use (typically curved in Maritime industry).
- Once compiled the graph can be used to determine values given one aspect of the ordered pairs.
- Graphs are used for a variety of purposes in the Maritime industry; including stability, working load limits, fuel consumption, pump rates and engine performance.

1.2 Graphs: Solving Simultaneous Equations

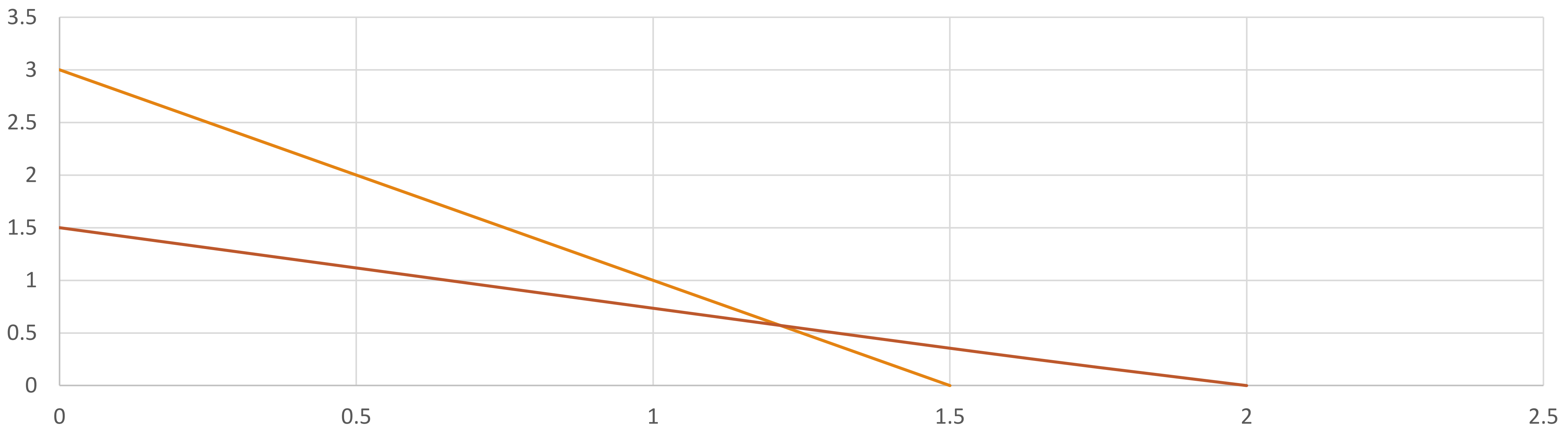
- You can use graphs to solve simultaneous equations by plotting the corresponding point on the axis as the other half of the ordered pair equals 0.
- The slope is given by the factor applied to the other part of the ordered pair.
- The correct answer is given by the intersect of the two.

So for example if: $2X + Y = 3$ and $X – 4Y = 2$

In the first equation if $X = 0$ then $Y = 3$, if $Y = 0$ then $-2X = 3$

In the second equation if $X = 0$ then $-4Y = 2$, if $Y = 0$ then $X = 2$

1.2 Graphs: Solving Simultaneous Equations



In the above example both lines are plotted by marking the point on the Y and X axis when the opposite coordinate is equal to zero.

The solution to the simultaneous equations can be determined by observing the intersect point (where the lines cross).

In this instance the answer is: X = 1.2 Y = 0.6

1.3 Proportion and Variation

- A key idea for knowing how things change in relation to each other is proportionality and variation.
- In math, proportion is the link between two or more ratios or fractions.
- Variation describes a relationship between two numbers which are related to each other.
- While proportion describes the relationship between two ratios that are related variation describes a relationship between two number that is expressed as a formula with factors.

Inverse Proportion/Variation): When one thing goes up, the other thing goes down at the same rate, and the other way around. The sum of the two numbers stays the same.

An inverse proportion is usually written as xy=k, where k is the ratio constant.

Direct Proportion (Variation): An increase or drop in two quantities happens at the same rate in direct proportion. If one number doubles, the other number also doubles, and if one number halves, the other number halves as well.

A direct proportion is usually written as y=kx, where k is the constant of proportionality.

1.3 Interpolation

- Interpolation is a way to figure out numbers that are between two or more known data points based on the data points that are available.
- It includes estimating what the value of a function or series will be at an intermediary point within the range of known data.
- In many areas, like math, statistics, engineering, computer science, and finance, interpolation is used to look at data, make models, and make predictions.
- Interpolation is a useful way to approximate intermediate values based on the data you have access to. It is used a lot in science and engineering where you need to make accurate guesses about intermediate values. But it's important to pick the right interpolation method and think about the assumptions and limits that come with each one.

1.3 Interpolation: Practice

Linear Interpolation:

Linear interpolation involves connecting two adjacent data points with a straight line relationship and estimating the value at an intermediate point along that line.

This method assumes a linear relationship between the data points and utilizes proportionality between the two ranges to determine estimates.

It’s primary use aboard is to interpolate the values from tank or stability tables.

Polynomial Interpolation:

Polynomial interpolation involves fitting a polynomial curve to the given data points and estimating the value at the desired point based on this curve.

The most common polynomial interpolation method is the Lagrange interpolation formula or Newton's divided difference formula.

1.3 Linear Extrapolation

- By stretching a linear trend, linear extrapolation is a mathematical method used to guess values that are outside the range of known data points.
- It includes guessing what a function or series will be worth at points outside the range of observed data based on the straight line relationship seen in the range of observed data. Linear extrapolation suggests that the variables have a relationship that is roughly linear and goes beyond the range of data that has been collected.
- It's important to be careful when extrapolating beyond the measured data range, even though linear extrapolation can give you useful information and predictions.
- Extrapolation works if the basic connection between the variables stays the same, which might not always be the case. Also, values that are extrapolated may not be as reliable and be subject to more uncertainty than values that are interpolated within the range of recorded data. Because of this, it is very important to think about the limits and assumptions of linear extension and be careful when interpreting the results.

1.4 Geometry

- The mathematics of geometry deals with the study of shapes, sizes, properties, and relationships of objects in space. It encompasses various branches, including Euclidean geometry, analytic geometry, differential geometry, and topology.

1.4 Geometry: Angles

Acute Angle:
- An angle that measures less than 90 degrees.
- An angle measuring 45 degrees is an acute angle.

Right Angle:
- An angle that measures exactly 90 degrees.
- The angle formed by the corner of a square is a right angle.

Obtuse Angle:
- An angle that measures greater than 90 degrees but less than 180 degrees.
- An angle measuring 120 degrees is an obtuse angle.

Straight Angle:
- An angle that measures exactly 180 degrees, forming a straight line.
- The angle formed by the hands of a clock at 6 o'clock is a straight angle.

Reflex Angle:
- An angle that measures greater than 180 degrees but less than 360 degrees.
- An angle measuring 270 degrees is a reflex angle.

Full Angle:
- An angle that measures exactly 360 degrees, forming a complete revolution.
- A circle consists of full angles.

1.4 Geometry: Shapes

Quadrilateral

If you have four sides, four vertices, and four angles, you have a quadrilateral. The sum of all the angles inside it is always 360 degrees. Different types of quadrilaterals, like parallelograms, trapeziums, rhombuses, rectangles, and squares, can be identified by their side lengths, angle measurements, and ability to be symmetrical.

Parallelogram

A parallelogram is a type of quadrilateral in which the lengths of the two opposite sides are equal and parallel. The angles on opposite sides of a parallelogram are also equal, and the diagonals cut through each other. Parallelograms are made up of shapes like rectangles, rhombuses, and squares, but each of them also meets other requirements.

Trapezium

There is at least one pair of parallel sides in a trapezium, which is a quadrilateral. When speaking British English, a trapezium doesn't have any parallel sides. But when speaking American English, the meaning is the opposite: a trapezium (also called a trapezoid) has at least one pair of parallel sides. The sections that are not parallel are called the legs, and the sections that are parallel are called the bases. If the lengths of the non-parallel sides are the same and the angles at the bases are the same, then the trapezium is isosceles.

Rhombus

For example, a rhombus is a parallelogram whose four sides are all the same length. A rhombus is like a parallelogram in that the two opposite sides are parallel, the two opposite angles are equal, and the two diagonals cut each other in half. A rhombus's diagonals are also straight and cut the angles of the shape in half. For a square to be a square, all of its angles must be 90 degrees.

1.4 Geometry: Triangles

Equilateral Triangle

A triangle is said to be equilateral if all three of its sides are the same length. In an equilateral triangle, this means that all three angles are the same length, 60 degrees each. An example of an equilateral triangle is a triangle whose sides all measure 5 units.

Isosceles Triangle

This type of triangle has at least two sides that are the same length. Because of this, the angles across from the equal sides are also equal. An isosceles triangle is one whose sides are all the same length, which is 4 units.

Scalene Triangle

In contrast to equilateral and isosceles triangles, a scalene triangle has all three sides of different lengths. Because of this, every angle in a scalene triangle is also unique. It's possible for the sides of a scalene triangle to be 3, 4, or 5 units long.

Right Triangle
A right triangle has one angle that is 90 degrees, which is called a right angle. The side that is not at a right angle is called the hypotenuse. The other two sides are called legs. A triangle with sides that are 3, 4, and 5 units long is an example of a right triangle.

Acute Triangle
All three angles in an acute triangle are less than 90 degrees, which means they are all acute. As an example, think about a triangle whose angles are 30, 60, and 90 degrees.

Obtuse Triangle
Unlike an acute triangle, an obtuse triangle has one angle that is greater than 90 degrees. One angle in this kind of triangle is obtuse, while the other two are acute. A triangle with angles of 45 degrees, 90 degrees, and 45 degrees could be used as an example.

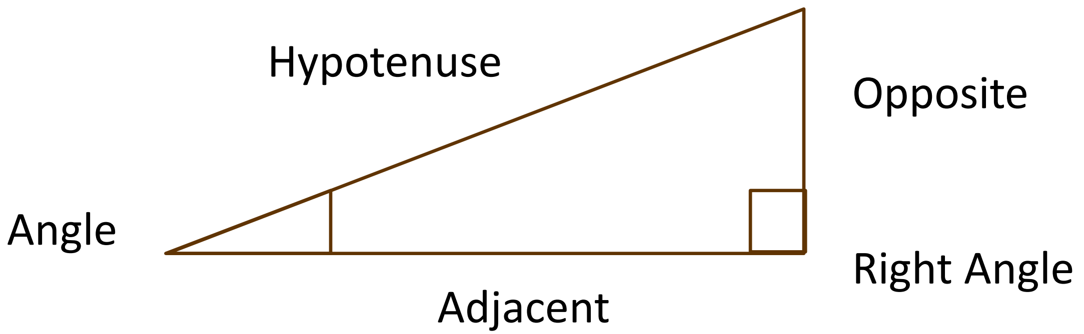
1.4 Geometry: Triangles Properties

Angle Sum Property

The sum of the inside angles of a triangle is always 180 degrees. This can be used to calculate any angle given you have the other two.

Pythagorean Theorem

In a right angle triangle, the square of the hypotenuse's length (the side across from the right angle) is equal to the sum of the squares of the other two sides' lengths.



1.4 Geometry: Parallel Lines

- Parallel lines are defined as lines that are equidistant from each other and never intersect. They lie in the same plane but do not meet regardless of how far they are extended.
- Corresponding angles formed by a transversal intersecting parallel lines are equal. Similarly, alternate interior angles and alternate exterior angles are also equal.
- Parallel lines have the same slope. If two lines have different slopes, they cannot be parallel.
- The distance between two parallel lines remains constant. This distance is measured as the perpendicular distance between any two points on the lines.
- When a transversal intersects two parallel lines, it forms pairs of equal angles: corresponding angles, alternate interior angles, and alternate exterior angles.
- If two lines are cut by a transversal and corresponding angles are equal, then the lines are parallel.

1.4 Geometry: Perpendicular Lines

- Perpendicular lines are two lines that intersect at a 90-degree angle, forming right angles.
- Perpendicular lines form right angles where they intersect. The measure of each angle formed is 90 degrees.
- The slopes of two perpendicular lines are negative reciprocals of each other.
- Perpendicular lines intersect at a single point, known as the point of intersection.
- The shortest distance between two intersecting lines occurs at the point of intersection. This distance is known as the perpendicular distance.
- A line perpendicular to a line segment at its midpoint is called a perpendicular bisector. It divides the line segment into two equal parts.
- If two lines intersect to form right angles, then they are perpendicular.

1.4 Geometry: Parts of a Circle

The center of a circle is a fixed point that serves as the point of origin for the circle. It is equidistant from all points on the circumference.

Circumference of a circle is the boundary or the outer edge of the circle. It is the distance around the circle.

The radius of a circle is the distance from the center of the circle to any point on the circumference. It is represented by the symbol

The diameter of a circle is the longest chord that passes through the center and connects two points on the circumference. It is equal to twice the radius.

A chord of a circle is a line segment joining two points on the circumference. A diameter is a special case of a chord that passes through the center.

1.4 Geometry: Formulas for Circles

Pi, or π, is a mathematical constant that is about the same as 3.14159.

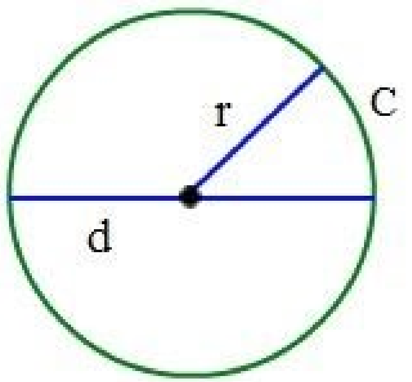
Diameter: The longest line that goes through the middle of a circle is its diameter.

Radius: The line that goes from the circle's center to any point on its edge is called its radius.

Circumference: The distance around a circle is called its circumference.

Area: The area of a circle is the space inside the circle's circumference.

Arc Length is determined as a proportion of circumference using angle of arc from center.



$$\text{Area } A = \pi r^2$$

$$\text{Circumference } C = 2\pi r$$

$$\text{Diameter } d = 2r$$

$$\text{Radius } r = \sqrt{\frac{A}{\pi}}$$

1.4 Geometry: Chords of a Circle

Definition: A chord is a line segment joining two points on the circumference of a circle. It serves as the shortest distance between two points on the circle.

Length: The length of a chord varies depending on the distance between the two points on the circumference. It can range from being equal to the diameter (when passing through the center) to shorter than the diameter.

Midpoint: The midpoint of a chord lies on the diameter of the circle, dividing the chord into two equal parts.

Perpendicular Bisector: A chord's perpendicular bisector passes through its midpoint and is perpendicular to it. This bisector divides the chord equally into two segments.

Secant: When extended to intersect the circle at two distinct points, a chord becomes a secant. It divides the circle into two segments: the minor segment and the major segment.

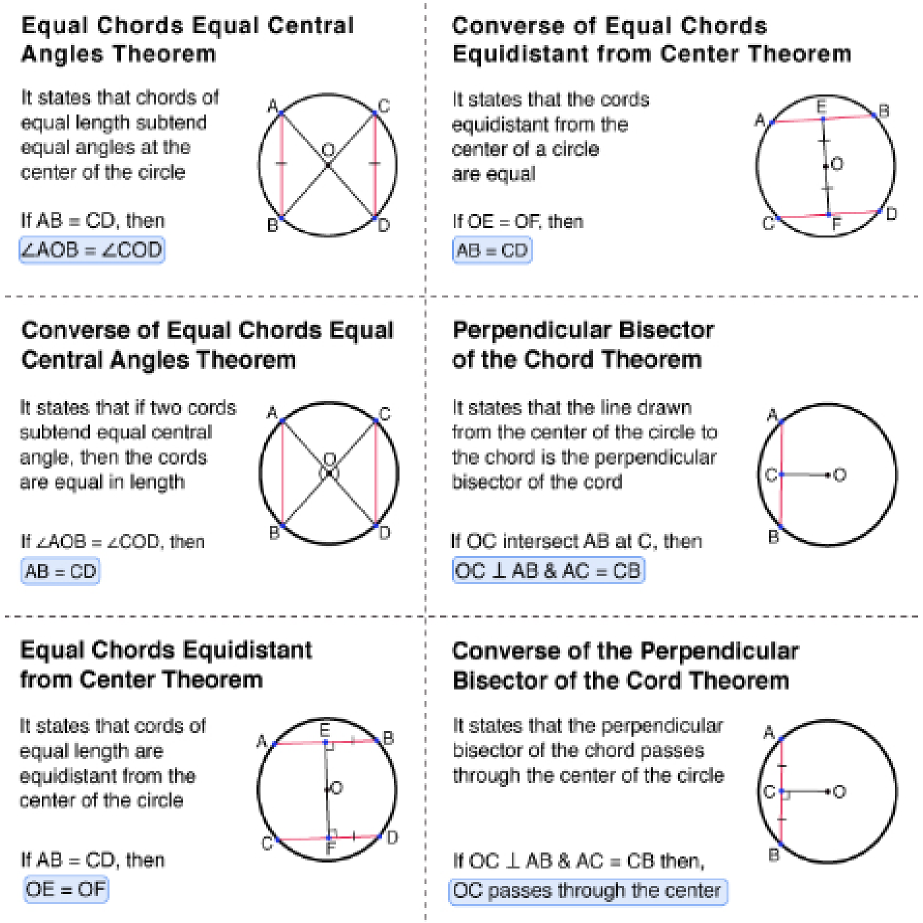
1.4 Geometry: Chords of a Circle

Length: The length of a chord can be determined using the Pythagorean theorem if the distance between the two points on the circumference and the radius of the circle are known.

Perpendicular Bisector: The perpendicular bisector of a chord passes through the circle's center, dividing the chord into two equal parts and being perpendicular to the chord.

Angle with Tangent: The angle between a chord and a tangent to the circle at one of the chord's endpoints equals the angle subtended by the chord at the center of the circle.

Angle Inscribed by Chord: The angle subtended by a chord at the circle's center is twice the angle subtended by the same chord at any point on the circumference.



1.4 Geometry: Areas of a Segment and Sector

Sector

A circle's sector is the area bounded by two radii and the arc that goes between them.

Segment

A circle segment is the area bounded by an arc and a chord that connects the arc's ends. First, figure out the area of the sector (as shown above), and then take away the area of the triangle made by the two radii and the chord to get the area of the segment.

You can use these formulas to find the area of any sector or segment of a circle as long as you know the radius of the circle and the size of the triangle's central angle.

$$\text{Area of a Sector} = \frac{n}{360} \times \pi r^2$$

where:

- n is the central angle in degrees,
- π is a mathematical constant approximately equal to 3.14159, and
- r is the radius of the circle.

$$\text{Area of a Segment} = \text{Area of a Sector} - \text{Area of Triangle}$$

$$\text{Area of Triangle} = \frac{1}{2}ab\sin(C)$$

$$\text{Area of a Segment} = \frac{r^2}{2}\left(\frac{\pi n}{180} - \sin n\right)$$

$360^\circ = 2\pi$ radians,

1.5 Trigonometry

- Trigonometry is a branch of math that looks at how the lengths of triangles' sides and angles relate to each other.
- It goes on to look at the properties of circles and wave functions, which makes it an important part of many fields, such as physics, engineering, astronomy, and architecture.

1.5 Trigonometry

Angles can be measured in degrees or radians, two primary units for describing the size of an angle.

Degrees

The number 360 equal parts that make up a full rotation around a circle is used to measure an angle. There is one degree (°) in each part. This division is made up on the spot and is based on old astronomy. You can also use minutes and seconds to measure angles. One minute is equal to sixty seconds, and one degree is equal to sixty minutes.

Radians

It makes more sense to use radians as a unit of angle measurement in math and science. This term is based on the diameter of a circle. A circle's radius is equal to the angle made by an arc whose length is equal to the circle's radius.

$$360^\circ = 2\pi \text{ radians},$$

1.5 Trigonometry: Sine, Cosine and Tan

The three main trigonometric functions are sine, cosine, and tangent. They connect the angles of a right triangle to the ratios of two of its sides.

Sin (sin)

To find the sine of an angle in a right triangle, divide the length of the hypotenuse by the length of the other side. It shows the y-coordinate of a point on the unit circle.

Cosine (cos)

The cosine of an angle is the ratio of the length of the side next to it to the length of the hypotenuse in a triangle with a right angle. It shows the x-coordinate of a point on the unit circle.

Tangent (tan)

If you compare the sine and cosine of an angle, you get the tangent of that angle. This is the same thing as comparing the opposite side of a right-angled triangle to the side next to it.

These functions are very important in trigonometry because they let you find triangle angles and side lengths when you know other numbers. They can also be used to show periodic events in the form of sine and cosine waves, which are not just right-angled triangles.

1.5 Trigonometry: Sine, Cosine and Tan

Sine, Cosine and Tangent

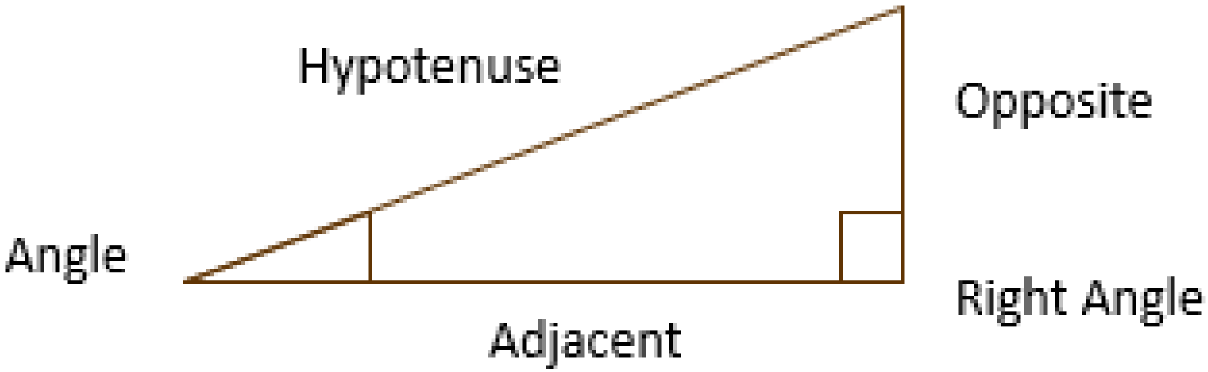
When referring to a right angle triangle the cosine, sine or tangent of the triangle's angle is equal to the result of two sides divided against one another.

This inverse variability is best remembered using the acronym **SOHCAHTOA**

Sine = Opposite/Hypotenuse

Cosine = Adjacent/Hypotenuse

Tangent = Opposite/Adjacent



1.6 Mensuration

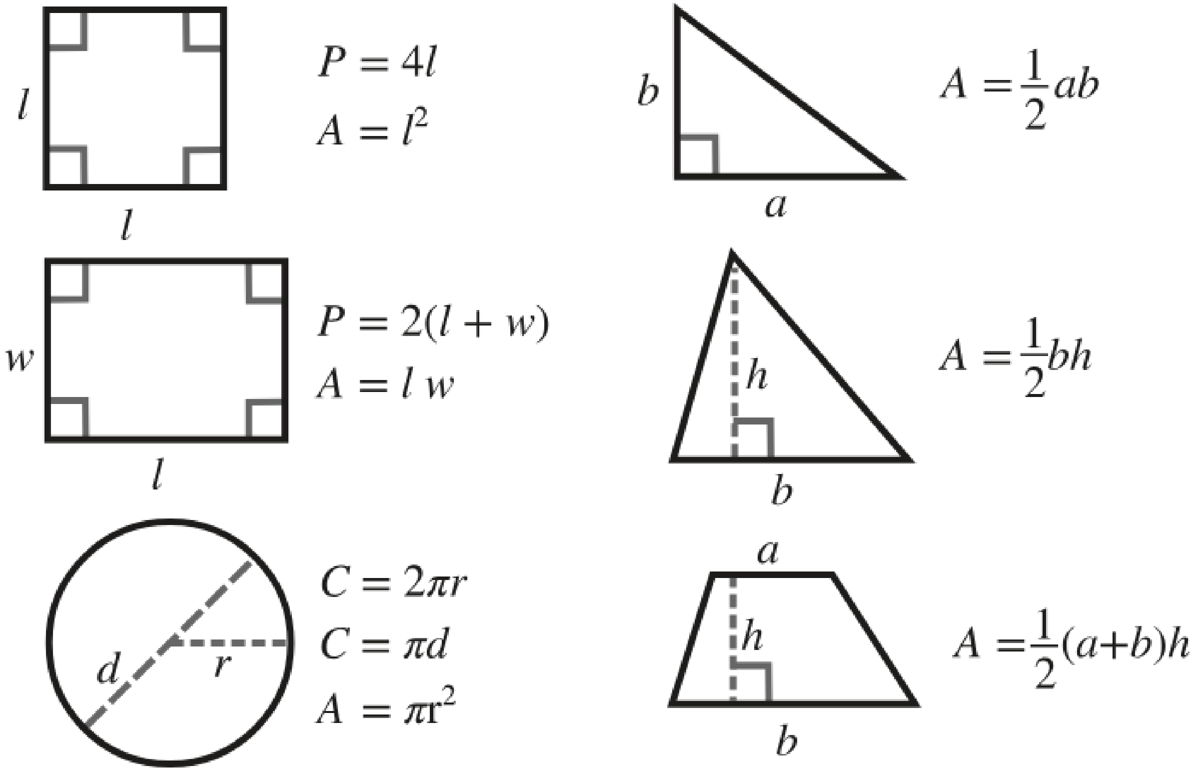
- Mensuration is a branch of mathematics that studies how to measure length, area, and volume, among other things, of geometric shapes and figures.
- To find the sizes of two-dimensional (2D) shapes like squares, rectangles, circles, and triangles, as well as three-dimensional (3D) objects like cubes, cylinders, spheres, and pyramids, algebraic equations and geometric formulas are used.
- Calculating the perimeter and area of shapes involves recognizing the shape type and applying the corresponding formula.

1.6 Mensuration: Perimeter

- The length of all of a geometric shape's edges is its perimeter.
- It is a linear measure, which means that it is measured in units like inches, meters, centimeters, and so on. The idea of perimeter can be used for any polygon, like triangles, rectangles, squares, and other odd shapes. It can also be used for the circumference of circles and other curved shapes.
- The perimeter of a shape is found by adding up the lengths of all of its sides.
- The perimeter of a circle is called the circumference and is found by the formula 2 x (pie x radius).

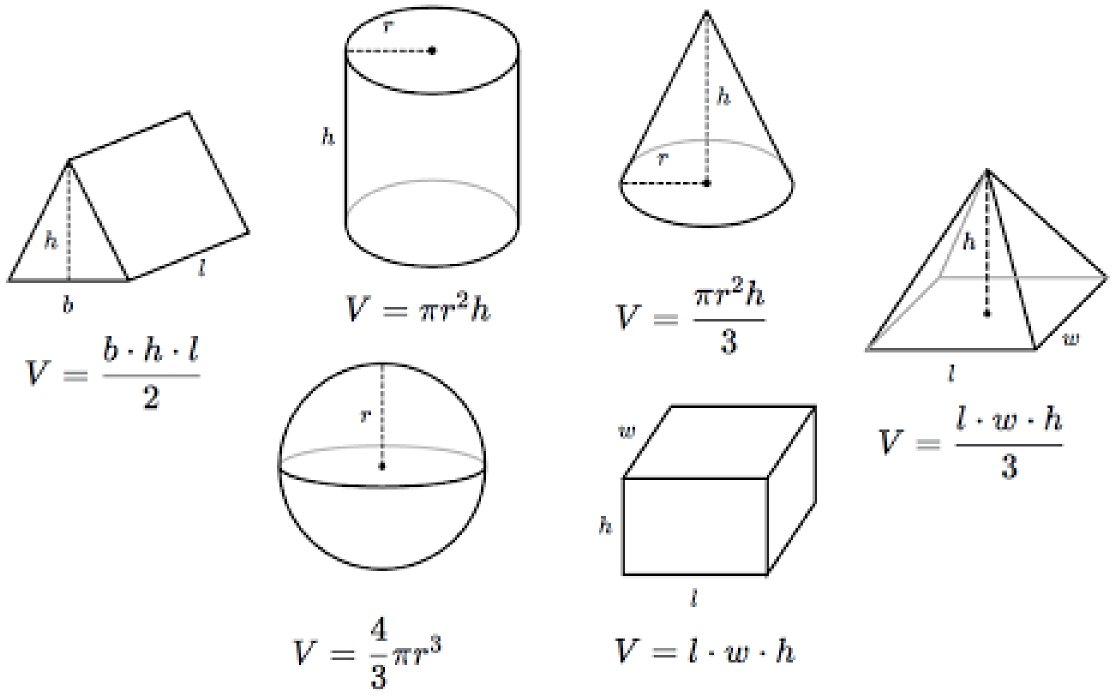
1.6 Mensuration: Area

- The area is a way to measure the space inside the edges of a two-dimensional shape.
- It is written in terms of square units, like square meters, square centimeters, and square inches.
- The shape affects the formula used to find the area.



1.6 Mensuration: Volume

- Volume is a way to measure how much space something takes up in three dimensions.
- It measures how much solid matter something can hold and is given in cubic units like liters, cubic meters, and so on.
- Volume is important to know in many fields, from medicine to manufacturing, because it tells professionals how much space something takes up or how much material they need to make something.



1.7 Spherical Triangle

- A spherical triangle is a shape on the surface of a sphere made up of three big circle arcs. These arcs are where the sphere meets planes that go through its center.
- In fields like astronomy, geophysics, and navigation, spherical triangles are used to figure out distances and directions on the globe.
- The angles of a spherical triangle add up to more than 180 degrees but less than 540 degrees. This is because the sphere's curvature changes the shape of the triangle.
- The sides of a spherical triangle are measured by the angles they subtend at the center of the sphere. These are usually given in degrees or radians. Imagine they are all arcs on the outside of a sphere measured in degrees from the center of that sphere.
- The area of a spherical triangle on a sphere of radius r is found by the equation $(A+B+C-\pi)r\ 2$
- Where A, B, and C are the internal angles of the triangle in radians, and π is the mathematical constant.

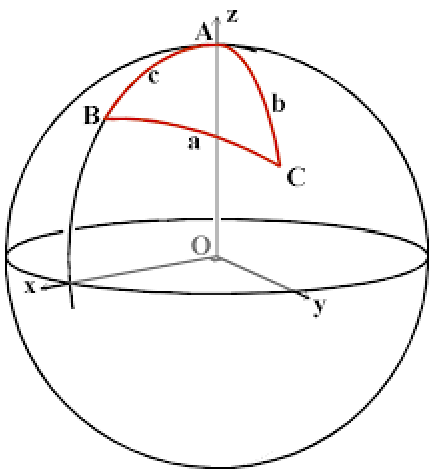
1.7 Spherical Triangle

- These functions and laws are very important for solving spherical triangles because they let you figure out unknown sides and angles when you know other parts of the triangle.
- They are very important for tasks that need to know exactly where they are on Earth's surface and how to move across it, like navigation, which needs to find the shortest path between two points on the globe (great circle distance) or figure out the bearing from one point to another.

Spherical Law of Sines: For a spherical triangle with angles A, B, and C, and opposite sides a, b, and c respectively, $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$.

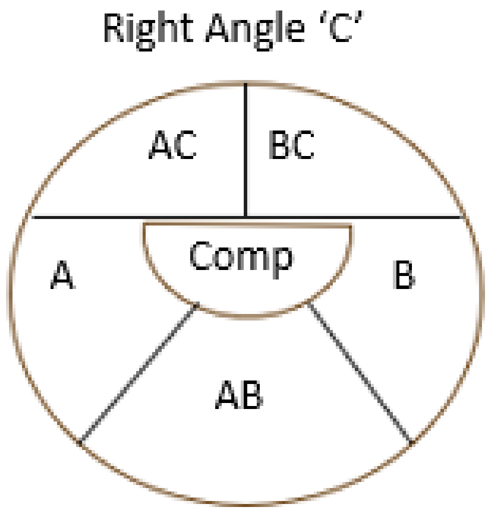
Spherical Law of Cosines for Sides: For each side a of the spherical triangle, $\cos a = \cos b \cos c + \sin b \sin c \cos A$, where A is the angle opposite side a.

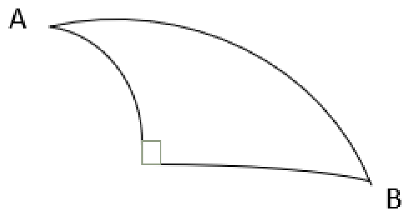
Spherical Law of Cosines for Angles: Relates the cosines of the angles of a spherical triangle to the cosines of the sides and the cosine of the other angle, $\cos A = -\cos B \cos C + \sin B \sin C \cos a$.



1.7 Spherical Triangle: Napier's Rules

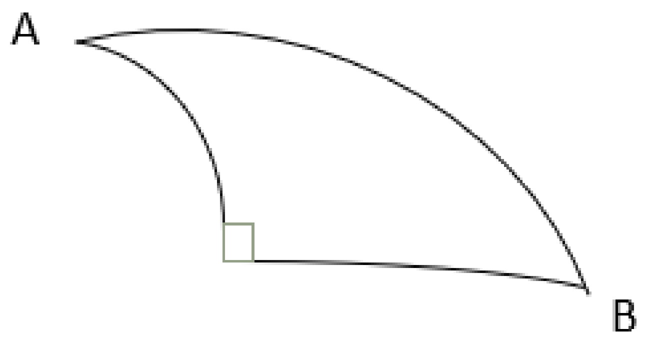
- Napier's rule is a set of mnemonic rules that were created by the Scottish mathematician John Napier in the 1600s.
- A right angled spherical triangle is a spherical triangle (on the surface of a sphere) where one of the angles is exactly 90 degrees.
- With Napier's rules you must first illustrate the right angled spherical triangle's angles and sides by building a circular diagram with the right angle marked at the top as shown.
- The parts marked as complement in the diagram are equal to the angle taken from 90 degrees.

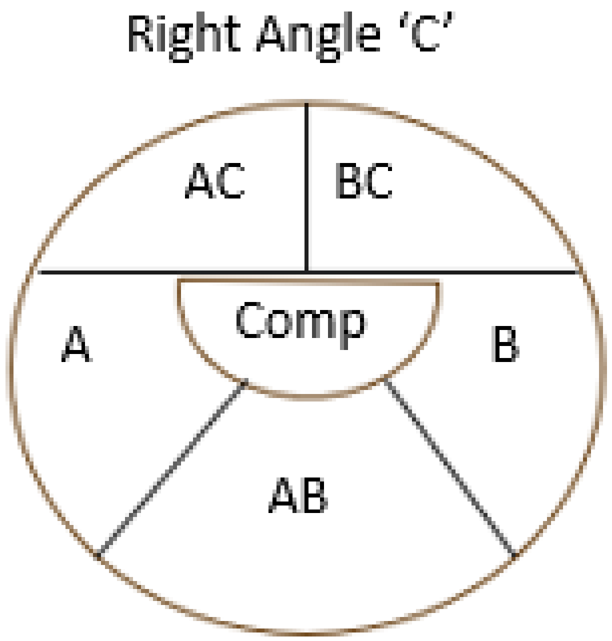




1.7 Spherical Triangle: Napier’s Rules

- The middle part's sine is equal to the sum of the tangents of the parts adjacent to it.
- The sine of the middle part is equal to the sum of the cosines of the two opposite parts.
- If the sum of angles and lines utilized is over 180 degrees than answer is deducted from 180 degrees.





1.8 Vectors

- Vectors are mathematical symbols used to show things that have both size and direction.
- They are very important in many fields, like physics, engineering, and computer science, because they are used to talk about physical things like force, speed, displacement, and acceleration.
- Its magnitude tells you how long it is.
- The direction that tells you what way it is pointing or what angle it is at.
- Angles, components, or unit vectors are just a few of the ways that this direction can be described.

Graphical Representation

Represent the first vector as a directed line segment on a coordinate plane, starting from the origin. Then, represent the second vector as another directed line segment starting from the head of the first vector.

Vector Subtraction

To find the difference between two vectors graphically, imagine the second vector being reversed in direction. This is equivalent to adding the negative of the second vector to the first vector. The resultant vector is the vector from the tail of the first vector to the head of the "reversed" second vector.

1.9 Circle, Ellipse and Hyperbola

Circle

A circle is made up of points in a plane that are all the same distance from the center. Radius is the measure of how far any point on the circle is from the center.

The center of the circle is the same distance from every point on its edge. When you multiply the radius by 2, you get the diameter. Around their center, circles are equal on both sides.

Ellipse

An Ellipse is an enclosed arc connected from both ends to the center of a circle.

The longest diameter of the ellipse, which goes through the foci, is called the major axis.

Along with being perpendicular to the major axis, the minor axis has the smallest diameter.

Hyperbola

Any point on a hyperbola is always farther away from two fixed points, called foci, than any other point on the hyperbola. This is called an absolute value difference.

The line that goes through the hyperbola's points is called the transverse axis.

Along with going through the middle of the hyperbola, the conjugate axis is also perpendicular to the transverse axis.

In short, the distance from the center of a circle to its edge stays the same.

Ellipses have two fixed points called foci, and the distances from any point to the foci always add up to the same amount.

Hyperbolas have two fixed points called foci, and the distances from any point to the foci are always different from each other.

1.10 Statistics

Mathematics has a subject called statistics that studies how to gather, sort, analyze, interpret, and present data. It includes ways to gather, summarize, and analyze numerical data so that people can make smart choices in business, economics, engineering, science, and other fields.

Random Experiments: A random experiment is a method or process that can have one of several possible outcomes, and the outcome cannot be known for sure ahead of time. For example, you could flip a coin, roll a die, or do a survey.

What is central tendency? Central tendency is the measure that shows where the middle or center of a set of data is. The mean, median, and mode are all common ways to find the central tendency. The median is the middle value when the data is sorted, the mean is the average value of the set, and the mode is the value that shows up most often.

Measures of Dispersion: These measures tell you how spread out or variable a dataset is. The range, the variance, and the standard deviation are all common ways to measure dispersion. The range is the difference between the worst and best values. The variance and standard deviation show how far each data point is on average from the mean.

Distributions that are always the same: These show how the odds of continuous random variables are spread out. The normal distribution, the uniform distribution, and the exponential distribution are some examples. Instead of discrete probability mass functions, these distributions are shown by probability density functions.

1.10 Statistics

Correlation and Regression: Correlation checks how strong and in what direction two variables are connected. You can measure it with the correlation coefficient, which goes from -1 to 1. The relationship between a dependent variable and one or more independent variables is looked into in regression analysis. A linear equation is used to model this relationship in linear regression, and multiple linear regression adds more variables to the model to make it more general.

Straight Line Regression: This is a way to use statistics to show how two or more variables that are not dependent on each other are related. It thinks that the variables are related in a straight line and uses the least squares method to guess the parameters of the regression equation.

Multiple Linear Regression: Multiple linear regression adds more variables to linear regression, letting it work with more than one. It shows how the dependent variable is related to two or more independent variables. This lets you study how different factors affect the dependent variable.

Method of Least Squares: This is a statistical method used in regression analysis to get a rough idea of what the parameters of a regression model are. It finds the line or curve that fits the data the best by minimizing the sum of the squared differences between the values that were observed and the values that the model predicted.

To find out how accurate the regression model's predictions are, you can look at the standard error of the regression estimate. It gives a number to the variation of the data points around the regression line. If the standard error is lower, it means that the model fits the data better.